Name: ______________________ Block: __________ Teacher: _______________

Algebra 1

Unit 3A Notes: Quadratic Functions - Factoring and Solving Quadratic Functions and Equations

DISCLAIMER: We will be using this note packet for Unit 3A. You will be responsible for bringing this packet to class EVERYDAY. If you lose it, you will have to print another one yourself. An electronic copy of this packet can be found on my class blog.

Standard	Lesson
Write expressions in equivalent forms to solve problems **MGSE9–12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
MGSE9–12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression.	
MGSE9–12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.	
Interpret structure of expressions **MGSE9-12.A.SSE.2** Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	
Create equations that describe numbers or relationships **MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic functions.	
MGSE9-12.A.CED.2 Create quadratic equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)	
MGSE9–12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Example: Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.	
Solve equations and inequalities in one variable **MGSE9-12.A.REI.4** Solve quadratic equations in one variable.	
MGSE9–12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from $ax^2 + bx + c = 0$.	
MGSE9–12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).	

Unit 3A: Factoring & Solving Quadratic Equations

After completion of this unit, you will be able to…

Learning Target #1: Factoring
- Factor the GCF out of a polynomial
- Factor a polynomial when $a = 1$
- Factor a polynomial when $a \neq 1$
- Factor special products (difference of two squares)

Learning Target #2: Solving by Factoring Methods
- Solve a quadratic equation by factoring a GCF.
- Solve a quadratic equation by factoring when a is not 1.
- Create a quadratic equation given a graph or the zeros of a function.

Learning Target #3: Solving by Non Factoring Methods
- Solve a quadratic equation by finding square roots.
- Solve a quadratic equation by completing the square.
- Solve a quadratic equation by using the Quadratic Formula.

Learning Target #4: Solving Quadratic Equations
- Solve a quadratic equation by analyzing the equation and determining the best method for solving.
- Solve quadratic applications

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Timeline for Unit 3A

Monday	Tuesday	Wednesday	Thursday	Friday
January 27th	**January 28th** Day 1- Factoring Quadratic Expressions – GCF	**29th** Day 2 - Factoring Quadratic Trinomials, $a = 1$	**30th** Day 3 - Factoring Quadratic Trinomials, $a \neq 1$	**31st** Day 4 - Factoring Special Products **Quiz – Factoring Quadratics**
February 3rd Day 5a – Solving Quadratics (GCF, $a = 1$, $a \neq 1$)	**4th** Day 5b – Solving Quadratics (GCF, $a = 1$, $a \neq 1$)	**5th** Day 6a – Solving By Taking Square Roots	**6th** Day 6b – Solving By Taking Square Roots	**7th** Day 7a - Solving by Completing the Square
10th Day 7b - Solving by Completing the Square	**11th** Day 8 - Solving by Quadratic Formula	**12th** Review Solving Quadratics **Quiz – Solving Quadratics**	**13th** Unit 3A Test Review	**14th** **Unit 3A Test**

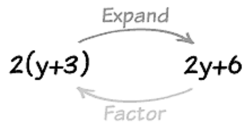
Day 1 – Factor by GCF

Standard(s):
MGSE9–12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression.

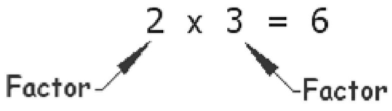
What is Factoring?

Factoring

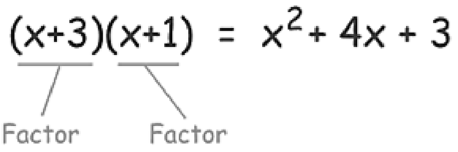
- Finding out which two expressions you ______________ together to get one single expression.
- "Splitting" an expression into a product of simpler expressions.
- The opposite of expanding or distributing.



Numbers have factors:



Expressions have factors too:



Review: Finding the GCF of Two Numbers

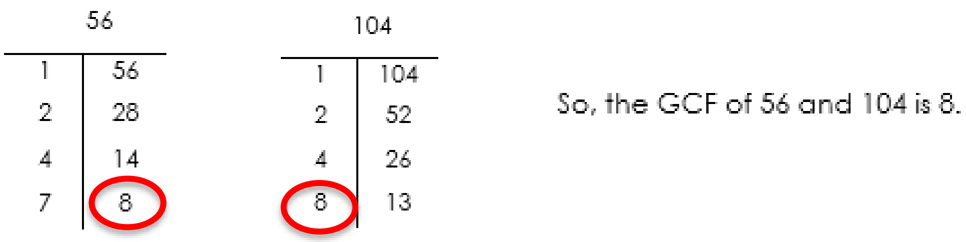
Common Factors

- Factors that are shared by two or more numbers

Greatest Common Factor (GCF)

- To find the GCF create a factor t-chart for each number and find the largest common factor

Example: Find the GCF of 56 and 104



56	
1	56
2	28
4	14
7	8

104	
1	104
2	52
4	26
8	13

So, the GCF of 56 and 104 is 8.

Practice: Find the GCF of the following numbers.

a. 30, 45

b. 12, 54

Finding the GCF of Two Expressions

To find the GCF of two expressions, create a factor chart for the two numbers AND expand the variables. Circle what is common to both.

Example: Find the GCF of $36x^2y$ and $16xy$

Practice: Find the GCF of the following pairs of expressions.

1) $15x^3$ and $9x^2$

2) $9a^2b^2$, $6ab^3$, and $12b$

3) $8x^2$ and $7y^3$

Factoring by GCF

Steps for Factoring by GCF

1. Find the greatest common factor of all the terms.

2. The GCF of the terms goes on the outside of the expression and what is leftover goes in parenthesis after the GCF.

3. After "factoring out" the GCF, the only that number that divides into each term should be 1.

Practice: Factor each expression.

1) $x^2 + 5x$ **GCF =**

2) $x^2 – 8x$ **GCF =**

3) $28x - 63$ **GCF =**

4) $18x^2 – 6x$ **GCF =**

5) $-2m^2 – 8m$ **GCF =**

6) $-9a^2 - a$ **GCF =**

7) $6x^3 – 9x^2 + 12x$ **GCF =**

8) $4x^3 + 6x^2 – 8x$ **GCF =**

9) $15x^3y^2 + 10x^2y^4$ **GCF =**

Day 2 – Factor Trinomials when a = 1

Standard(s):
MGSE9–12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression.

Quadratic Trinomials

2nd Degree
(Quadratic)

3 Terms
(Trinomial)

$$ax^2 + bx + c$$

Factoring a trinomial means finding two ____________ that when multiplied together produce the given **trinomial**.

Skill Preview: "Big X" Problems

Complete the diamond problems. The top cell contains the product of the numbers in the left and right cells, while the bottom cell contains the sum.

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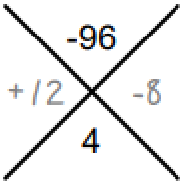

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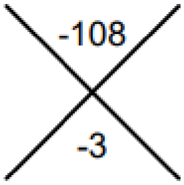

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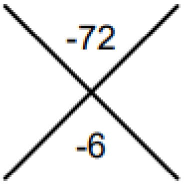

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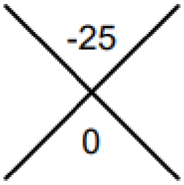

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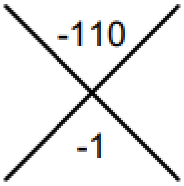

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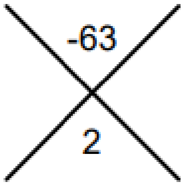

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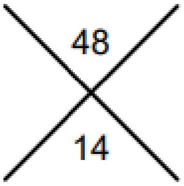

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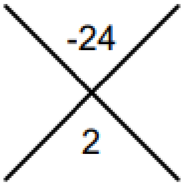

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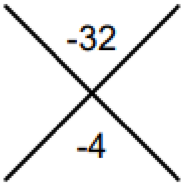

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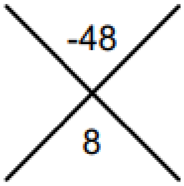

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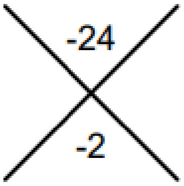

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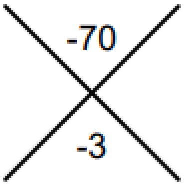


Factoring using Quadratic Trinomials when a = 1

Steps for Factoring when a = 1

Step 1: ALWAYS check to see if you can factor out a GCF

Step 2: Draw parentheses for the binomial factors and fill in the variables. Ex (x   ) (x   )

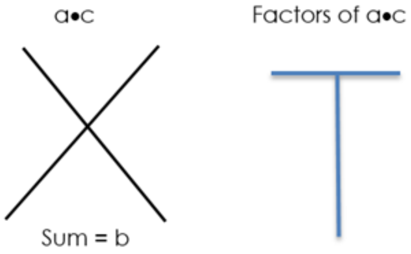
Step 3: Complete a "Big X" and T-chart

Step 4: Determine what two numbers can be multiplied to get your "a·c" term and added to get your "b" term. (Use a factor T-chart)

Step 5: Fill the factors in the parentheses.

Example: Factor the trinomial.

$x^2 + 6x + 8$



factored form: ______________________________

Factor the following trinomials.

a. Factor $x^2 + 4x – 32$

b. Factor $x^2 – 3x – 18$

c. Factor $x^2 – 36$

d. Factor $2x^2 + 16x + 24$

Remember: You must ALWAYS include the GCF on the outside of the factored form!

Day 3 – Factor Trinomials with a ≠1

In the previous lesson, we factored polynomials for which the coefficient of the squared term, "a" was always 1. Today we will focus on examples for which a ≠ 1.

Looking for Patterns

What do you observe in the following Area Models?

	$3x$	$+4$
x	$3x^2$	$+4x$
-3	$-9x$	-12

$(x-3)(3x+4) = 3x^2 - 5x - 12$

	$4x$	$+3$
$2x$	$8x^2$	$+6x$
$+1$	$+4x$	$+3$

$(2x+1)(4x+3) = 8x^2+10x+3$

Factoring is the ____________________of distributing or multiplying.

STEP 1: - ALWAYS check to see if you can factor out a GCF.	**Factor:** $2x^2 - 5x + 3$
STEP 2: - Complete a "Big X" and T-chart - Determine what two numbers can be multiplied to get your "a·c" term and added to get your "b" term.	 
STEP 3: - Create a 2x2 Area Model and place your original "a" term in the top left box and "c" term in the bottom right box. - Fill the remaining two boxes with the two numbers you found in "Big X" and place an x after them.	Factored Form: ___________________________
STEP 4: - Factor out a GCF from each row and column	
STEP 5: - Check your factors on the outside by multiplying them together to make sure you get all the expressions in your box.	

Factoring a ≠ 1

Using the Area Model. Factor the following trinomials.

1. $5x^2 + 14x - 3$ Factored Form: ________________________

---

2. $2x^2 - 17x - 30$ Factored Form: ________________________

---

3. $12x^2 + 56x + 64$ Factored Form: ________________________

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4. $6x^2 - 40x + 24$ Factored Form: ________________________

Remember: You must ALWAYS include the GCF on the outside of the factored form!

Day 4 – Factor Special Products

Standard(s): MGSE9-12.A.SSE.2
Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Review: Factor the following expressions:

a. $x^2 - 49$ b. $x^2 - 25$ c. $x^2 - 81$

1. What do you notice about the "a" term? ____________________

2. What do you notice about the "c" term? ____________________

3. What do you notice about the "b" term? ____________________

4. What do you notice about the factored form? ____________________

The above polynomials are a special pattern type of polynomials; this pattern is called a

Difference of Two Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Always subtraction
Both terms are perfect squares
Always two terms

Can you apply the "Difference of Two Squares" to the following polynomials?

a. $9x^2 - 49$ b. $9x^2 - 100$ c. $4x^2 - 25$ d. $16x^2 - 1$

e. $x^2 + 25$ f. $25x^2 - 64$ g. $36x^2 - 81$ h. $49x^2 - 9$

Review: Factor the following expressions:

a. $x^2 + 8x + 16$ b. $x^2 - 2x + 1$ c. $x^2 - 10x + 25$

1. What do you notice about the “a” term? ____

2. What do you notice about the “c” term? ____

3. What do you notice about the “b” term? ____

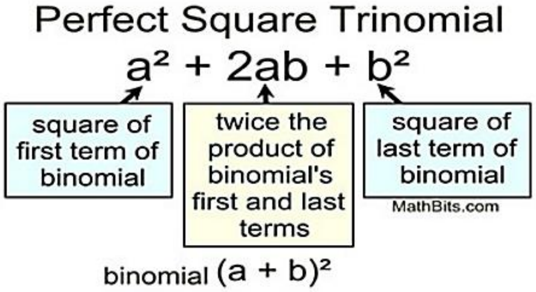
4. What do you notice about the factored form? ____

The above polynomials are a second type of pattern; this pattern type is called a

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$



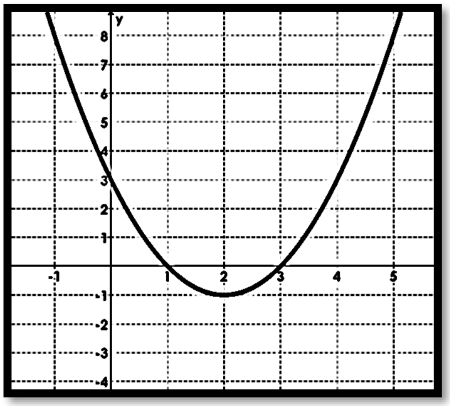
Using the perfect square trinomial pattern, see if you can fill in the blanks below:

a. $x^2 +$ ______ $+ 36$ b. $x^2 -$ ______ $+ 81$ c. $x^2 -$ ______ $+ 64$

d. $x^2 + 4x +$ ______ e. $x^2 - 6x +$ ______ f. $x^2 + 20x +$ _______

Day 5 – Solving Quadratics (GCF, when a = 1, when a not 1)

Standard(s):
MGSE9-12.A.REI.4 Solve quadratic equations in one variable.



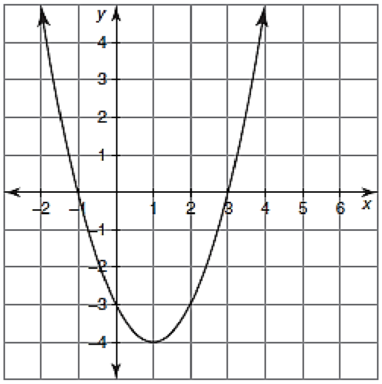
The Main Characteristics of a Quadratic Function

- A quadratic function always has an exponent of ______.
- The standard form of a quadratic equation is ____
- The U-shaped graph is called a ___________________.
- The highest or lowest point on the graph is called the __________.
- The points where the graph crosses the x-axis are called the ___________________.
- The points where the graph crosses are also called the ________________________ to the quadratic equation. A quadratic equation can have ________, ________, or ________ solutions.

In this unit, we are going to explore how to **solve quadratic equations**.

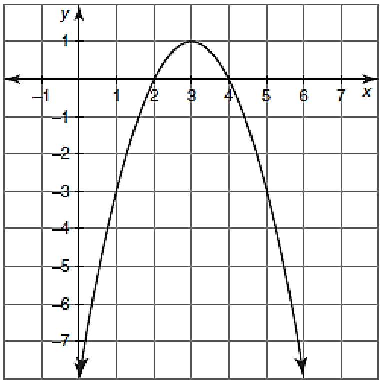
Solving a quadratic equation really means:
Finding its ____________, ____________, ____________ or _____________.

Create an equation to represent the following graphs:



Zeros: x = _____ & _____

y = __________________



Zeros: x = ____ & ____

y = __________________

Zero Product Property and Factored Form

Zero Product Property

- The ***zero-product property*** is used to _______________ an equation when one side is zero and the other side is a product of binomial factors.
- The zero product property states that if $a \cdot b = 0$, then $a = 0$ or $b = 0$

Examples: Identify the zeros of the functions:

a. $(x – 2)(x + 4) = 0$

b. $x(x + 4) = 0$

c. $(x + 3)^2 = 0$

d. $y = (x + 4)(x + 3)$

e. $y = x(x – 9)$

f. $f(x) = 5(x – 4)(x + 8)$

Solve the following quadratic equations by factoring (GCF) and using the Zero Product Property.

1: Factoring & Solving Quadratic Equations - GCF

Practice: Solve the following equations by factoring out the GCF.

1. $3x^2 = 18x$

Factored Form: ________________

Zeros: ________________________

2. $-3x^2 – 12x = 0$

Factored Form: __________________

Zeros: _________________________

Solve the following quadratic equations by factoring and using the Zero Product Property.

2: Factoring & Solving Quadratic Equations when a =1

3. $y = x^2 - 6x + 9$

Factored Form: ________________

Zeros: ______________________

4. $x^2 + 4x = 32$

Factored Form: ________________

Zeros: ______________________

3: Factoring & Solving Quadratic Equations when a not 1

5. $y = 5x^2 + 14x - 3$

Factored Form: ________________________

Zeroes: ________________________

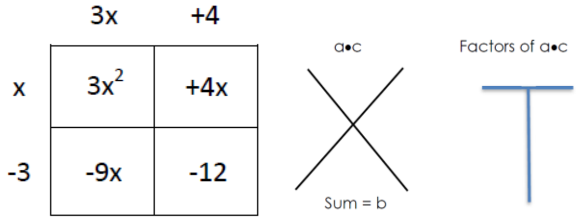
6. $2x^2 - 8x = 42$

Factored Form: ________________________

Zeroes: ________________________

Graphic Organizer: Reviewing Methods for Factoring

Before you factor any expression, you must always check for and factor out a **Greatest Common Factor (GCF)!**

	Looks Like	**How to Factor**	**Examples**
GCF (Two Terms)	$ax^2 - bx$	Factor out what is common to both terms (mentally or list method)	$x^2 + 5x = x(x + 5)$ $18x^2 – 6x = 6x(3x – 1)$ $-9x^2 – x = -x(9x + 1)$
A = 1	$x^2 + bx + c$	Think of what two numbers multiply to get the c term and add to get the b term (Think of the diamond). You also need to think about the signs: $x^2 + bx + c = (x + \#)(x + \#)$ $x^2 – bx + c = (x - \#)(x - \#)$ $x^2 – bx – c/x^2 + bx – c = (x + \#)(x - \#)$	$x^2 + 8x + 7 = (x + 7)(x + 1)$ $x^2 – 5x + 6 = (x – 2)(x – 3)$ $x^2 – x – 56 = (x + 7)(x – 8)$
A not 1	$ax^2 + bx + c$	**Area Model:** $3x^2 - 5x - 12$  **Factored Form :** (x – 3) (3x + 4)	$9x^2 – 11x + 2 = (9x – 2) (x – 1)$ $2x^2 + 15x + 7 = (2x + 1)(x + 7)$ $3x^2 – 5x – 28 = (2x + 7)(x – 4)$
Difference of Two Squares	$x^2 – c$	Both your "a"and "c" terms should be perfect squares and since there is no "b" term, it has a value of 0. You must also be subtracting the a and c terms. Your binomials will be the exact same except for opposite signs. **Difference of Squares** $a^2 – b^2 = (a + b)(a – b)$	$x^2 – 9 = (x + 3)(x – 3)$ $x^2 – 100 = (x + 10)(x – 10)$ $4x^2 – 25 = (2x + 5)(2x – 5)$
Perfect Square Trinomials	$x^2 + bx + c$ "c" is a perfect square "b" is double the square root of c	Factor like you would for when a = 1	$x^2 – 6x + 9 = (x – 3)(x – 3)$ $= (x – 3)^2$ $x^2 + 16x + 64 = (x + 8)(x + 8)$ $= (x + 8)^2$

Day 6 – Solving Quadratics by Finding Square Roots

Standard(s): MGSE9–12.A.REI.4b
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).

Review: If possible, simplify the following radicals completely.

a. $\sqrt{25}$

b. $\sqrt{125}$

c. $\sqrt{24}$

Explore: Solve the following equations for x:

a. $x^2 = 16$

b. $x^2 = 4$

c. $x^2 = 9$

d. $x^2 = 1$

Remember: When taking square roots to solve for x, you get a positive and negative answer!

Steps for Solving Quadratics by Finding Square Roots

1. Add or Subtract any constants that are on the same side of x^2.
2. Multiply or Divide any constants from x^2 terms. "Get x^2 by itself"
3. Take square root of both sides and set equal to positive and negative roots (±).

Ex: $x^2 = 25$
$\sqrt{x^2} = \sqrt{25}$
$x = \pm 5$
$x = +5$ and $x = -5$

REMEMBER WHEN SOLVING FOR X YOU GET A ____________ AND ____________ ANSWER!

Solve the following for x:

1) $x^2 = 49$

2) $x^2 = 20$

3) $x^2 = 0$

4) $3x^2 = 108$

5) $x^2 - 11 = 14$

6) $7x^2 - 6 = 57$

7) $4x^2 - 6 = 74$

Solving by Finding Square Roots (More Complicated)

Steps for Solving Quadratics by Finding Square Roots with Parentheses

1. Add or Subtract any constants outside of any parenthesis.
2. Multiply or Divide any constants around parenthesis/squared term. "Get ($)^2$ by itself"
3. Take square root of both sides and set your expression equal to BOTH the positive and negative root (±). Ex: $(x + 4)^2 = 25$

$$\sqrt{(x + 4)^2} = \sqrt{25}$$
$$(x + 4) = \pm 5$$
$$x + 4 = +5 \text{ and } x + 4 = -5$$
$$x = 1 \text{ and } x = -9$$

4. Add, subtract, multiply, or divide any remaining numbers to isolate x.

REMEMBER WHEN SOLVING FOR X YOU GET A POSITIVE AND NEGATIVE ANSWER!

Solve the following for x:

1) $(x-4)^2 = 81$

2) $(p-4)^2 = 16$

3) $10(x-7)^2 = 440$

4) $\frac{1}{2}(x+8)^2 = 14$

5) $-2(x+3)^2 - 16 = -48$

6) $3(x-4)^2 + 7 = 67$

Day 7 – Solving by Completing the Square

Standard(s): MGSE9–12.A.REI.4b
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).

Some trinomials form special patterns that can easily allow you to factor the quadratic equation. We will look at two special cases:
Review: Factor the following trinomials.

1. $x^2 – 6x + 9$	2. $x^2 + 10x + 25$	3. $x^2 – 16x + 64$

(a) How does the constant term in the binomial relate to the b term in the trinomial?

(b) How does the constant term in the binomial relate to the c term in the trinomial?

Problems 1-3 are called **Perfect Square Trinomials**. These trinomials are called perfect square trinomials because when they are in their factored form, they are a binomial squared.

An example would be $x^2 +12x + 36$. Its factored form is $(x + 6)^2$, which is a binomial squared.

But what if you were not given the c term of a trinomial? How could we find it?

Complete the square to form a perfect square trinomial and then factor.

a. $x^2 + 12x + \square$ b. $z^2 - 4z + \square$ c. $x^2 - 18x + \square$

Solving equations by "COMPLETING THE SQUARE" $\boxed{}$

The Equation: $x^2+6x+2=0$

STEP 1: Write the equation in the form

$x^2 + bx + \boxed{} = c + \boxed{}$

(Bring the constant to the other side)

$$x^2+6x+\underline{\boxed{}}=-2+\boxed{}$$

STEP 2: Make the left-hand side a perfect square trinomial by adding $\left(\frac{b}{2}\right)^2$ to **both** sides

$$x^2+6x+\boxed{(3)^2}=-2+\boxed{(3)^2}$$

STEP 3: Factor the left side, simplify the right side

$$(x+3)^2=7$$

STEP 4: Solve by taking square roots on both sides

$$x+3=\sqrt{7} \quad \text{and} \quad x+3=-\sqrt{7}$$

$$x=\sqrt{7}-3 \quad \text{and} \quad x=-\sqrt{7}-3$$

Group Practice: Solve for x by "Completing the Square".

1. $x^2 - 6x - 72 = 0$

X = ______________________________

2. $x^2 + 80 = 18x$

X = ______________________________

3. $x^2 - 14x - 59 = -20$

X = ______________________________

4. $2x^2 - 36x + 10 = 0$

X = ______________________________

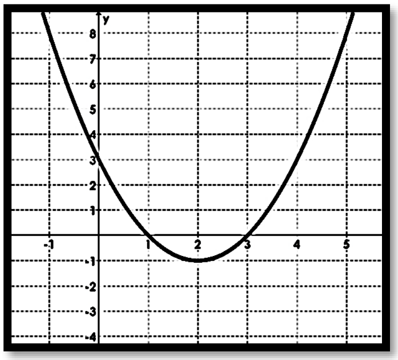
Day 8 - Solving by Quadratic Formula

Standard(s): MGSE9–12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).

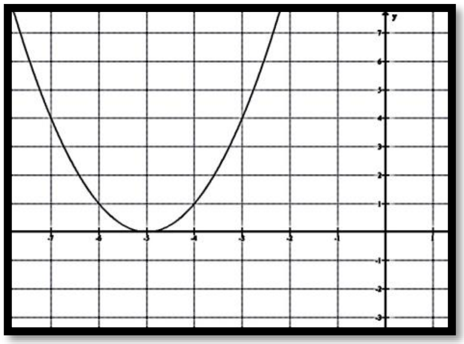
Exploring the Nature of Roots

Determine the number of real solutions (roots/x-intercepts) for the following graphs:

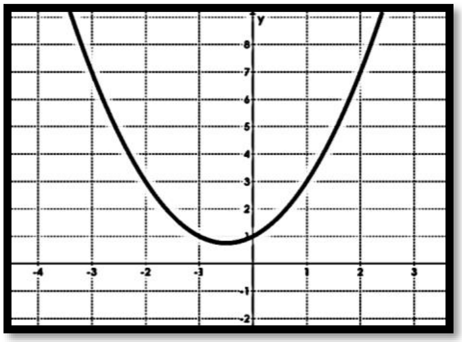
1. $f(x) = x^2 - 4x + 3$



2. $f(x) = x^2 + 10x + 25$



3. $f(x) = x^2 + x + 1$



The Discriminant

Given a quadratic function in standard form: $ax^2 + bx + c = 0, \text{ where } a \neq 0$,

- The **discriminant** is found by using: ***b² – 4ac***
- The discriminant can be used to determine the real number of solutions for a quadratic equation.

Interpretation of the Discriminant ($b^2 - 4ac$)

- If $b^2 - 4ac$ is positive:
- If $b^2 - 4ac$ is zero:
- If $b^2 - 4ac$ is negative:

Practice: Find the discriminant for the previous three functions:

a) $f(x) = x^2 - 4x + 3$ b) $f(x) = x^2 + 10x + 25$ c) $f(x) = x^2 + x + 1$

Discriminant: _______ Discriminant: _______ Discriminant: _______

#of real solutions: _____ #of real solutions: _____ #of real solutions: _____

The Quadratic Formula

We have learned **three methods** for solving quadratics:

- **Factoring** *(Only works if the equation is factorable)*
- **Taking the Square Roots** *(Only works when equations are not in Standard Form)*
- **Completing the Square** *(Only works when a is 1 and b is even)*

What method do you use when your equations are not factorable, but are in standard form, and a may not be 1 and b may not be even?

The Quadratic Formula

for equations in standard form: $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x represents the zeros and $b^2 - 4ac$ is the discriminant

Practice with the Quadratic Formula

For the quadratic equations below, use the quadratic formula to find the solutions. Write your answer in simplest radical form.

1) $\mathbf{4x^2 - 13x + 3 = 0}$ a = ____ b = ____ c = _____

2) $9x^2 + 6x + 1 = 0$ a = ____ b = ____ c = _____

Discriminant: _________________

Solutions: ____________________

Discriminant: _________________

Zeros: ________________________

3) $7x^2+8x+3=0$ a = ____ b = ____ c = _____

4) $-3x^2+2x=-8$ a = ____ b = ____ c = _____

Discriminant: ____________

X = ______________________

Discriminant: ____________

Roots: __________________

Determining the Best Method

Non-Factorable Methods		
Completing the Square	**Finding Square Roots**	**Quadratic Formula**
$ax^2 + bx + c = 0$, *when a = 1 and b is an even #* **Examples** $x^2 – 6x + 11 = 0$ $x^2 – 2x - 20= 0$	*$ax^2 - c = 0$* Parenthesis in equation **Examples** $2x^2 + 5 = 9$ $5(x + 3)^2 – 5 = 20$ $x^2 – 36 = 0$	*$ax^2 + bx + c = 0$* Any equation in standard form Large coefficients **Examples** $3x^2 + 9x – 1 = 0$ $20x^2 + 36x – 17 = 0$

Factorable Methods	
A = 1 & A Not 1 (Factor into 2 Binomials)	**GCF**
$ax^2+ bx + c = 0$, when a =1 *$ax^2 \pm bx \pm c = 0$, when a > 1* *$x^2 - c = 0$* **Examples** $3x^2 – 20x – 7 = 0$ $x^2 – 3x + 2 = 0$ $x^2 + 5x = -6$ $x^2 – 25 = 0$	*$ax^2 + bx = 0$* **Examples** $5x^2 + 20x = 0$ $x^2 – 6x = 8x$