Slide 8

Warm-Up: EOC-Type Question

Louisa wants to memorize the names and atomic numbers of all 118 elements of the periodic table before the Pasteur High School Chemistry Challenge. On her first day of studying, Louisa memorizes the names and atomic numbers of the 6 noble gases: Helium (2 - He), Neon (10 - Ne), Argon (18 - Ar), Krypton (36 - Kr), Xenon (54 - Xe), and Radon (86 - Rn). Each subsequent day, Louisa plans to memorize the names and atomic numbers of 3 more elements than the number of elements she had memorized during the previous day.

A. Write an explicit equation and a recursive process to describe the sequence represented by the number of elements that Louisa studies each day.

$a_1 = 6 \quad d = 3$

$a_n = 6 + (n-1)3$

$a_n = 6 + 3n - 3$

Recursive Rule

$a_n = a_{n-1} + d$

Explicit equation: $a_n = 3n + 3$

Recursive process: $a_n = a_{n-1} + 3$

B. If Louisa continues to use this strategy to study, how many more elements will she memorize on her sixth day of studying as compared to her third day of studying? Show all work and justify your answer.

$a_6 = 3(6) + 3 = 21$

$a_3 = 3(3) + 3 = 12$

$a_6 - a_3 = 21 - 12 = 9$

Louisa will memorise 9 more elements on her 6th day of studying than on her third day.

4/20/2021 Unit 4

Day 6 - Arithmetic & Geometric Sequences

Essential Question

How can I write an explicit and recursive formula for geometric and arithmetic sequences?

Arithmetic Vs.Geometric Sequences

Note: Arithmetic Sequences are Linear Functions, while Geometric Sequences are Exponential Functions.

Arithmetic	Geometric
Add or Subtract by the same number (common difference)	Multiply by the same number (constant ratio)
Explicit: $a_n = a_1 + (n - 1)d$	Explicit: $a_n = a_1 \cdot r^{n-1}$
Recursive: $a_n = a_{n-1} + d$	Recursive: $a_n = r(a_{n-1})$

Explicit Formula

Why We Have a Formula for Sequences

Take a look at the following pattern: **4, 8, 12, 16, ...**

What is the 3rd term? ________ What is the 5th term? ________ What is the 7th term? _______

What is the pattern? ______________________________________ What is the 1st term? _______

What is the 54th term? _______ (You don't want to add ___ over and over 54 times?!?!?!?)

This is why the **Explicit Formula** was created – as long as you know your common difference and 1st term, you can create a rule to describe any arithmetic sequence and use it to find any term you want.

Explicit Formula: $a_n = a_1 + (n-1)d$

nth term 1st term term position common difference

Steps in Creating an Explicit Rule

Given the sequence: 4, 8, 12, 16,...	
1. Write down the Explicit Formula	$a_n = a_1 + (n-1)d$
2. Substitute the first term for a_1 and the common difference for *d*.	$a_1 = 4$ $d = 8 - 4 = 4$ $a_n = 4 + (n-1)4$
3. Simplify the right side of the equation so that the equation looks like $\boxed{y = mx + b}$ $\boxed{(a_n = dn + c)}$	$a_n = 4 + (n-1)4$ $a_n = 4 + 4n - 4$ $\boxed{a_n = 4n}$
4. To find the nth term, substitute the term number you want to find for ***n***.	$a_7 = 4(7)$ $a_7 = 28$

Practice $a_n = a_1 + (n-1)d$

Write an Explicit Rule for the following sequences:

a. 1, 8, 15,...

$a_1 = 1$

$d = 8-1 = 7$

$a_n = 1 + (n-1)7$

$a_n = 1 + 7n - 7$

$\boxed{a_n = 7n - 6}$

b. 4, 0, -4,...

$a_1 = 4$

$d = 0-4 = -4$

$a_n = 4 + (n-1)-4$

$a_n = 4 - 4n + 4$

$\boxed{a_n = -4n + 8}$

c. -5, 3, 11,...

$a_1 = -5$

$d = 3-(-5) = 8$ or $11-3=8$

$a_n = -5 + (n-1)8$

$a_n = -5 + 8n - 8$

$\boxed{a_n = 8n - 13}$

Practice: You do

Slide 10

2. The odometer on a car reads 60,473 on Day 1. Every day, the car is driven 54 miles. If this pattern continue what is the odometer reading on Day 20?

Type: Arithmetic

Explicit Formula: $a_{20} = 60,473 + (20-1)54$

Solution: 61,499

$d = 54$

$a_1 = 60,473$

$a_n = a_1 + (n-1)d$

Practice: You do

3. To package and ship an item, it costs $5.75 for the first pound and $0.75 for each additional pound. What is the cost of shipping of 12 pound package?

$a_1 = 5.75$ $d = 0.75$

Type: Arithmetic

Explicit Formula: $a_{12} = 5.75 + (12-1)0.75$

Solution: $a_{12} = \$14$

4. The table shows a car's value for 3 years after it is purchased. The values form a geometric sequence. How much will the car be worth in the 10th year?

Year	Value ($)
1	10,000
2	8,000
3	6,400

$r = \frac{8000}{10000} = 0.8$

$a_1 = 10{,}000$

Type: Geometric

Explicit Formula: $a_{10} = 10000(0.8)^{10-1}$

Solution: $a_{10} = \$1{,}342.18$

Arithmetic Sequences

Recursive Formula

The recursive formula allows you to find the next term in a sequence if you know the common difference and any term of the sequence.

$$a_n = a_{n-1} + d$$

Nth Term Previous Term Common Difference

Arithmetic Sequences

Example:

Use the formula to generate the first four terms of the sequence:

$$\begin{cases} a_1 = 10 \\ a_n = a_{n-1} + 3 \end{cases}$$

common difference

previous term

10, 13, 16, 19

Recursive Formula

For the following recursive formulas, find the first five terms:

1. $a_1 = 4$

   $a_n = a_{n-1} + 4$ — d

   $a_2 = 4 + 4 = 8$

   $a_3 = 8 + 4 = 12$

   $a_4 = 12 + 4 = 16$

   $a_5 = 16 + 4 = 20$

2. $a_1 = -7$

   $a_n = a_{n-1} - 6$

   $a_2 = -7 - 6 = -13$

   $a_3 = -13 - 6 = -19$

   $a_4 = -19 - 6 = -25$

   $a_5 = -25 - 6 = -31$

3. $a_1 = -3.5$

   $a_n = a_{n-1} + 9$

   $a_2 = -3.5 + 9 = 5.5$

   $a_3 = 5.5 + 9 = 14.5$

   $a_4 = 14.5 + 9 = 23.5$

   $a_5 = 23.5 + 9 = 32.5$

MC Practice Test

a_1 d

MC Practice

1. In the sequence above, the first term is 4 and each term after the first is 7 more than the previous term. What is the 12th term of the sequence?

$$a_n = a_1 + (n-1)d$$

   a. 77
   b. 81
   c. 84
   d. 86

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2. Find the 25th term of the sequence 7, 11, 15, 19, 23, ...

$d = 4 \quad a_1 = 7$

$a_{25} = 7 + (25-1)4$

   a. 103
   b. 104
   c. 107
   d. 111

---

3. Which represents the nth term of this sequence? 31, 36, 41, 46, 51, ...

$a_1 = 31 \quad d = 5$

   a. $51+(n-1)6$
   b. $51+(n-1)5$
   c. $31+(n-1)6$
   d. $31+(n-1)5$

---

4. What is the 9th term in this sequence? 20, 14, 8, 2,...

$d = -6$

$a_9 = 20 + (9-1)-6$

a. 62
b. -4
c. -22
d. -28

---

5. What are the first four terms in the sequence whose nth term is $a_n = (-2)^n + 1$

$a_1 = (-2)^1 + 1 = -1$

$a_2 = (-2)^2 + 1 = 5$

$a_3 = (-2)^3 + 1 = -7$

a. 3, 4, 5, 6
b. -1, 1, -1, 1
c. -1, 5, -7, 17
d. -2, 4, -8, 16

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6. The 8th term of an arithmetic sequence is 36. If the common difference is 2, what is the first term in the sequence?

$a_8 = a_1 + (7)2$

$36 = a_1 + \cancel{14}$

$-14 \qquad - \cancel{14}$

$22 = a_1$

a. 22
b. 24
c. 38
d. 64

Use your knowledge of sequences to answer the following multiple-choice questions.

1. The formula $\begin{cases} a_1 = 3000 \\ a_n = 0.80a_{n-1} \end{cases}$ can be used to model which scenario?

a.	The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.	c.	The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
b.	A bank account starts with a deposit of \$3000, and each year it grows by 80%.	d.	The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

2. At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the nth term of this sequence is $a_n = 25000 + 1000(n-1)$. Which rule best represents the equivalent recursive formula?

a.	$a_1 = 25000; a_n = 1000a_{n-1}$	c.	$a_1 = 25000; a_n = a_{n-1} + 1000$
b.	$a_1 = 1000; a_n = 25000a_{n+1}$	d.	$a_1 = 25000; a_n = a_{n+1} + 1000$

3. Which function represents this sequence?

n	1	2	3	4	5	...
a_n	−1	1	3	5	7	...

$d = 3 - 1 = 2$

a.	$a_n = a_{n-1} + 1$	c.	$a_n = 2a_{n-1}$
b.	$a_n = a_{n-1} + 2$	d.	$a_n = 2a_{n-1} - 3$

4. A theater has more seats in the back rows than it has in the front rows. At a particular theater each row has two more seats than the row in front of it. Which formulas model this situation if the front row has twenty seats?

a.	$a_n = a_{n-1} + 2$ and $a_n = 2n + 20$	c.	$a_n = 2a_{n-1}$ and $a_n = 2n + 20$
b.	$a_n = a_{n-1} + 2$ and $a_n = 2n + 18$	d.	$a_n = 2a_{n-1}$ and $a_n = 2n + 18$

$a_1 = 20 \quad d = 2$

Explicit

$a_n = 20 + (n-1)2$

$a_n = 20 + 2n - 2$

$\boxed{a_n = 2n + 18}$

Recursive

$\boxed{a_n = a_{n-1} + 2}$

5. Select TWO of the following statements that are TRUE based on the following pictorial sequence.

3 5 7 9

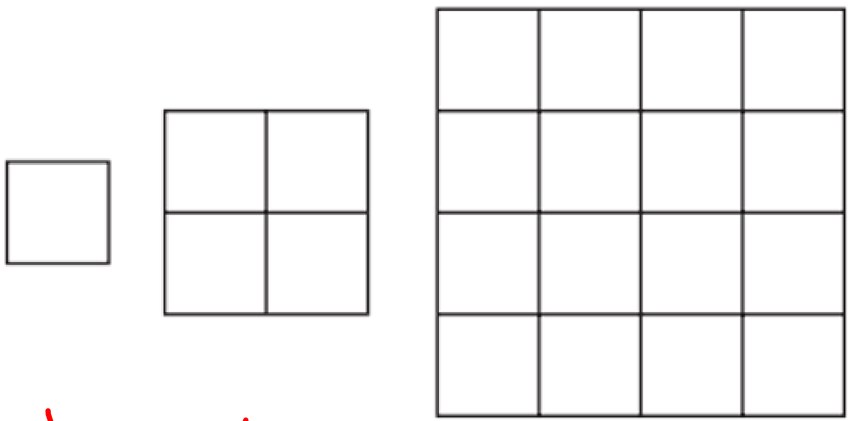
$a_1 = 3 \quad d = 2 \quad a_n = 3 + 2n - 2$

$a_n = 3 + (n-1)2 \quad \boxed{a_n = 2n + 1}$

a.	$a_n = 2a_{n-1}$		d.	$a_n = 3a_{n-1}$	
b.	$a_n = a_{n-1} + 2$	Recursive	e.	$a_n = 2n + 1$	Explicit
c.	$a_n = a_{n-1} - 2$		f.	$a_n = 2n + 3$	

More Patterns

1. Consider this pattern.



1 4 16

Which function represents the sequence that represents the pattern?

A. $a_n = (4)^{(n-1)}$

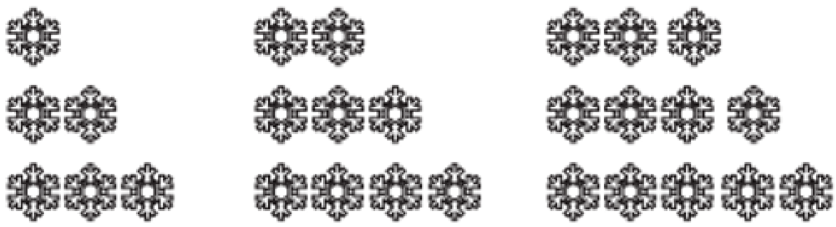
B. $a_n = (4)^{(a_n - 1)}$

C. $a_n = (a_n)(4)^{(n-1)}$

D. $a_n = (a_n)^4$

$a_n = a_1(r)^{n-1}$

2. **Consider this pattern.**



6 9 12

Which function represents the sequence that represents the pattern?

d = 3

A. $a_n = a_{n-1} - 3$
B. $a_n = a_{n-1} + 3$
C. $a_n = 3a_{n-1} - 3$
D. $a_n = 3a_{n-1} + 3$

Closing: Start working on Deltamath HW.

Any Questions?