Warm-Up 4/19/2021

Compound Interest

1. Janelle decided to invest money in an account that earns 3.5% compounded semi-annually. If she initially deposited $1,250.00 into the account and adds nothing to it, how much will it be worth in 5 years?

A. $1,484.61
B. $1,486.81
C. $1,363.27
D. $1,763.25

$$A = P(1+\frac{r}{n})^{nt}$$

$$A = 1250\left(1+\frac{0.035}{2}\right)^{2\times 5}$$

Growth & Decay

Growth: $y = a(1+r)^t$	Decay: $y = a(1-r)^t$

2. The population of a town is decreasing at a rate of 3% per year. In 2000, there were 1700 people. Write an exponential decay function to model this situation. Then find the population in 2012. $y=1700(0.97)^t$

$t = 2012 - 2000 = 12$

$y = 1700(0.97)^{12}$

$y = 1,179$ people

3. Russell's health and fitness blog is really taking off. The blog had 45,000 commenters this month and the number of commenters has consistently gone up by 10% per month. How many commenters can Russell expect to have in 5 months?

$y = 45,000(1+0.1)^5$

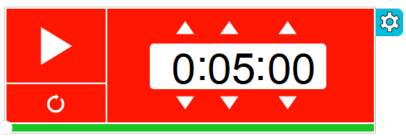
$y = 45000(1.1)$

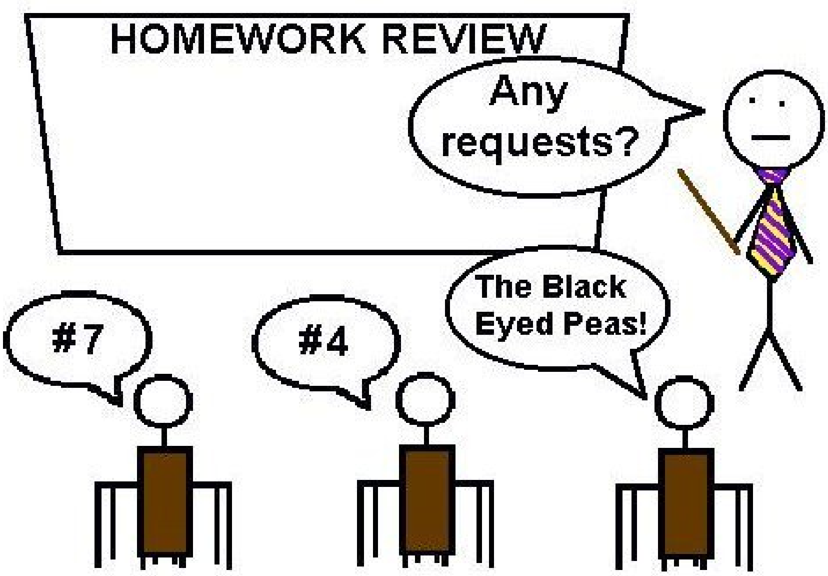
$y = 72,472$

Home Work Review 4/11/19

Reminder!!!

Deltamath and Digital Escape HW are due on Thursday, 4/21/2021





Essential Question 4/19/2021

How can I write an explicit formula for a geometric sequence?

Standard:

MGSE9-12.F.BF.2

Write geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect geometric sequences to exponential functions.

Geometric Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **geometric sequence** is a sequence in which you get the next consecutive term by multiplying or dividing a constant number called the **common ratio** or **constant ratio**.

Example: 5, 25, 125, 625, 3125...

What is the constant ratio? 5 What is the rule? x 5

Geometric sequences are considered exponential functions. The position of each term is called the **term number** or **term position**. We can think of the term number or position as the input (domain) and the actual term in the sequence as the output (range). Instead of using **x** for the **input**, we will use **n** and use a_n instead of using **y** for the **output**.

Input	Domain Term Number (n)	1	2	3	4	5
Output	Range Term (a_n)	1	6	36	216	1296

x6 x6 x6

Explicit Formula for Geometric Sequences

Explicit Formula:

$$\underbrace{a_n}_{\text{N}^{\text{th}}\text{ Term}} = \underbrace{a_1}_{1^{\text{st}}\text{ Term}} \cdot \overbrace{r}^{\text{Constant Ratio}}{}^{n-1}$$

Closed/Explicit:

- Can directly find nth term
- NOT required to list 1st term
- Used to find specific terms in a sequence - the Nth term. Example: the 54th term.

Steps in Creating an Explicit Rule

Given the sequence: 2, 4, 8, 16,...	
1. Write down the Explicit Formula	$a_n = a_1 \cdot r^{n-1}$
2. Substitute the first term for a_1 and the constant ratio for r.	$a_1 = 2 \quad r = \frac{4}{2} = 2$
3. To find the nth term, substitute the term number you want to find for ***n***.	$a_n = 2 \cdot 2^{n-1}$ $a_{14} = 2 \cdot 2^{14-1}$ $a_{14} = 2(2)^{13} = 16{,}384$

Practice: I do/We do

Write an Explicit Rule for the following sequences:

a. 3, 6, 12 ,...

$a_1 =$ 3

$r =$ $6/3 = 2$

Rule: $a_n = 3(2)^{n-1}$

b. 400, 200, 100, ...

$a_1 =$ 400

$r =$ $200/400 = 0.5$

Rule: $a_n = 400(0.5)^{n-1}$

Practice: You do

c. 40, 10, $\frac{5}{2}$, ...

$a_1 =$ 40

$r =$ $10/40 = 0.25$

Rule: $a_n = 40(0.25)^{n-1}$

d. -1, 3, -9, ...

$a_1 =$ -1

$r =$ $3/-1 = -3$

Rule: $a_n = -1(-3)^{n-1}$

e. 128, 32, 8, ...

$a_1 =$ 128

$r =$ $32/128 = 0.25$ or $1/4$

Rule: $a_n = 128(0.25)^{n-1}$

f. -2, -12, -72

$a_1 =$ -2

$r =$ $-12/-2 = 6$

Rule: $a_n = -2(6)^{n-1}$

Examples - Finding the Nth Term

Finding the Nth Term

To find the nth term, particularly when the nth term is quite large, you want to create an Explicit Rule first and then substitute that term number into the rule for n.

For the given sequences, create an explicit rule and then use the rule to find the following terms:

a. 1.5, 4.5, 13.5, ... $\underline{a_7}$

$a_1 = 1.5$

$r = \frac{4.5}{1.5} = 3$

$\boxed{a_n = 1.5(3)^{n-1}}$

$a_7 = 1.5(3)^{7-1}$

$a_7 = 1.5(3)^6$

$a_7 = 1{,}093.50$

b. 162, 108, 72, 48, ... $\underline{8^{th}\text{ term}}$

$a_1 = 162$

$r = 108/162 = 2/3$

$\boxed{a_n = 162(2/3)^{n-1}}$

$a_8 = 162(2/3)^7$

$a_8 = 9.\overline{481}$

Finding Terms Using an Explicit Rule

Practice **Finding Terms Using an Explicit Rule**

For the following sequences, find the first five terms:

a. $a_n = 3 \cdot 5^{n-1}$

$a_1 = 3(5)^{1-1} = 3$

$a_2 = 3(5)^1 = 15$

$a_3 = 3(5)^2 = 75$

$a_4 = 3(5)^3 = 375$

$a_5 = 3(5)^4 = 1{,}875$

b. $a_n = (-3)^{n-1}$

$a_1 = 1$

$a_2 = -3$

$a_3 = 9$

$a_4 = -27$

$a_5 = 81$

c. $a_n = -3 \cdot 2^{n-1}$

$a_1 = -3$

$a_2 = -6$

$a_3 = -12$

$a_4 = -24$

$a_5 = -48$

Recursive Rule

Recursive Formula

There is a second formula for arithmetic sequences called the **Recursive Formula.** The recursive formula allows you to find the next term in a sequence if you know the common difference and any term of the sequence.

$$a_1 = \text{first number}$$

$$\underbrace{a_n}_{\text{Nth Term}} = \underbrace{r}_{\text{Constant Ratio}}(\underbrace{a_{n-1}}_{\text{Previous Term}})$$

Recursive:

- Relates each term in the sequence to a previous term
- Must ALWAYS state 1st term

Examples - Finding Terms Using a Recursive Rule

Finding Terms Using a Recursive Formula

For the following recursive formulas, find the first five terms:

1. $a_1 = 4$
$a_n = 4(a_{n-1})$

2. $a_1 = -18$
$a_n = \frac{1}{3}(a_{n-1})$

3. $a_1 = 1025$
$a_n = \left(\frac{1}{5}\right)(a_{n-1})$

$a_2 = 4(4) = 16$
$a_3 = 4(16) = 64$
$a_4 = 4(64) = 256$
$a_5 = 4(256) = 1024$

$a_2 = -6$
$a_3 = -2$
$a_4 = -2/3$
$a_5 = -2/9$

$a_2 = 205$
$a_3 = 41$
$a_4 = 41/5$
$a_5 = 41/25$

Creating a Recursive Rule

Practice **Creating a Recursive Rule**

For the following sequences, create a recursive rule:

1. a. 2, 4, 8, 16,...

$a_1 = 2$

$r = 4/2 = 2$

$\boxed{a_n = 2(a_{n-1})}$

b. 4, 2, 1, .5,...

$a_1 = 4$

$r = 2/4 = 0.5$ or $1/2$

$\boxed{a_n = \frac{1}{2}(a_{n-1})}$

c. 6, 3, 1.5,...

$a_1 = 6$

$r = 3/6 = 1/2$

$\boxed{a_n = \frac{1}{2} \cdot a_{n-1}}$

d. 18, 54, 162,...

$a_1 = 18$

$r = 54/18 = 3$

$\boxed{a_n = 3 \cdot a_{n-1}}$

Practice: I do

Slide 5

2. a. Given a term and the common ratio, write the explicit formula: $a_5 = -64$, $r = 4$

$a_n = a_1(r)^{n-1}$

Rule

$\boxed{a_n = -0.25(4)^{n-1}}$

$a_5 = a_1(4)^{5-1}$

$-64 = a_1(4)^4$

$\frac{-64}{256} = \frac{\boxed{a_1}(256)}{256}$

$-0.25 = a_1$

Practice: You do

2. b. Given a term and the common ratio, write the explicit formula: $a_4 = 16$, $r = 2$

$a_n = a_1(r)^{n-1}$

$a_4 = a_1(2)^{4-1}$

Rule

$\boxed{a_n = 2(2)^{n-1}}$

$16 = a_1(2)^3$

$\frac{16}{8} = \frac{a_1 \cdot \cancel{8}}{\cancel{8}}$

$2 = a_1$

Practice: I do

3. a. Given two terms in a geometric sequence, write the explicit formula: $a_3 = 48$ and $a_4 = 192$

$$a_n = a_1(r)^{n-1}$$

$$r = \frac{192}{48} = 4$$

$$a_3 = a_1(4)^{3-1}$$

$$48 = a_1(4)^2$$

$$\frac{48}{16} = \frac{a_1 \cdot \cancel{16}}{\cancel{16}}$$

$$3 = a_1$$

Rule

$$\boxed{a_n = 3(4)^{n-1}}$$

Practice: You do

3. b. Given two terms in a geometric sequence, write the explicit formula: $a_4 = 250$ and $a_5 = 1250$

$$a_n = a_1(r)^{n-1}$$

$$r = \frac{1250}{250} = 5$$

$$a_4 = a_1(5)^{4-1}$$

1

$$250 = a_1(5)^3$$

$$\frac{250}{125} = \frac{a_1(\cancel{125})}{\cancel{125}}$$

$$2 = a_1$$

$$\boxed{\text{Rule: } a_n = 2(5)^{n-1}}$$

Practice: You do

4. The 10th term of a geometric sequence is 0.78125. The common ratio is 0.5. Find the first term of the sequence.

$$a_n = a_1(r)^{n-1}$$

$$a_{10} = a_1(r)^{10-1}$$

$$0.78125 = a_1(0.5)^9$$

$$\frac{0.78125}{0.001953125} = \frac{a_1(0.001953125)}{0.001953125}$$

$$400 = a_1$$

Practice: You do

5. It is time to call the exterminator! You found out that the number of termites under your house is tripling every week. If you have 8 termites on week 1, find the following:

$a_n = a_1(r)^{n-1}$

a. A sequence to show the growth of termites:

$a_n = 8(3)^{n-1}$

b. The number of termites after 12 weeks:

$a_{12} = 8(3)^{12-1}$

$a_{12} = 8(3)^{11}$

$a_{12} = 1{,}417{,}176$ termites

Practice: How would you do this?

6. A geometric sequence starts with the number 14 and the common ratio is 0.4. Colby finds that another number in the sequence is 0.057344. Which term in the sequence did Colby find?

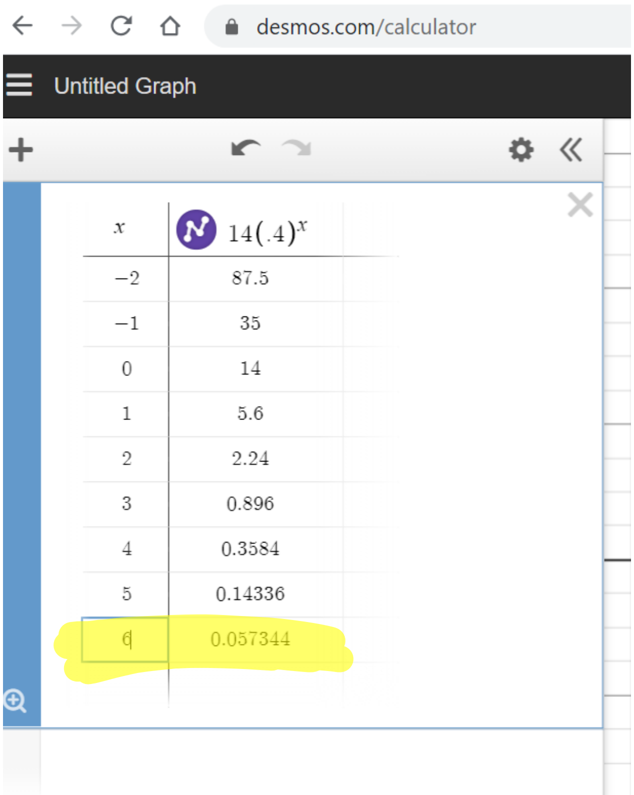
$$a_n = 14(0.4)^{n-1}$$

Calculator: $y = 14(0.4)^x$

look for the x-value that gives the y-value of 0.057344

$x = 6$

Term $= 6 + 1 = 7^{th}$ Term.



x	$14(.4)^x$
−2	87.5
−1	35
0	14
1	5.6
2	2.24
3	0.896
4	0.3584
5	0.14336
6	0.057344