Essential Question 4/15/2021

How can I use exponential functions to solve compound interest problems?

Standard:

MGSE9-12.A.CED.2

Create exponential equations in two or more variables to represent relationships between quantities. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

Compound Interest

- As you get older, you will come to learn a great deal about investing your money…savings accounts, stock market, mutual funds, bonds, etc. Today, we are going to learn about compound interest, which is a form of saving and earning money by letting it sit in an account over time.

- **Compound Interest** is interest earned or paid on both the principal and previously earned interest.

- In middle school, you learned about **simple interest**, which is interest that is only earned on the principal. It's formula is $I = Prt$, where P represents principal, r represents rate, t represents time, and I represents interest.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = balance after *t* years
P = Principal (original amount)
r = interest rate (as a decimal)
n = number of times interest is compounded per year
t = time (in years)

annually — n = 1
Semi-annually - n = 2
Monthly - n = 12
Quarterly - n = 4

Example 1

$A = P\left(1 + \frac{r}{n}\right)^{tn}$

Example 1: Write a compound interest function that models an investment of $1000 at a rate of 3% compounded quarterly. Then find the balance after 5 years.

P = 1000

r = 0.03

n = 4

t = 5

$A = 1000\left(1 + \frac{0.03}{4}\right)^{4 \times 5}$

$A = \$1,161.18$

Example 2

Example 2: Write a compound interest function that models an investment of $18,000 at a rate of 4.5% compounded annually. Then find the balance after 6 years.

P = 18,000

r = 0.045

n = 1

t = 6

$A = 18000\left(1 + \frac{0.045}{1}\right)^{6}$

$A = \$23,440.68$

n
annually = 1
Semi-annually = 2
monthly = 12
Quarterly = 4

Example 3

Example 3: Write a compound interest function that models an investment of $4,000 at a rate of 2.5% compounded monthly. Then find the balance after 10 years.

P = 4000

r = 0.025

n = 12

t = 10

$$A = 4000\left(1 + \frac{0.025}{12}\right)^{120}$$

$$A = \$5,134.77$$

Quick Check - 1

$y = a(1 \pm r)^t$

A pot of soup, currently at 84 C is left out to cool. If that temperature decreases by 5% per minute, what will the temperature be in 5 minutes?

$y = 84(1 - 0.05)^5$

$y = 64.99$

$y \approx 65C$

Quick Check - 2

The population of a small town started at 233 people in 1999. If the population grows at a rate of 16% per year, how many people are now in the town in 2006?

$y = 233(1+0.16)^7$

$t = 2006 - 1999 = 7$

$y = 233(1.16)^7$

$y = 658$ people

Growth & Decay Digital Escape Activity

Closing:

If there is a % change, the base for growth is ($1+r$) and for decay is ($1-r$).

What's going to be the base of the exponential function in the following cases?

0.2

1. 20% increase

$1+0.2$

$=1.2$

0.15

2. 15% decrease

$1-0.15$

$=0.85$

3. 8% increase

0.08

$1+0.08=1.08$

4. 4% decrease

0.04

$1-0.04=0.96$

5. 12.5% increase

0.125

$1+0.125$

$=1.125$

6. 42.5% decrease

0.425

$1-0.425$

$=0.575$