4/14/2021

Day 3 Notes – Applications of Exponential Functions – Growth/Decay

$y = ab^x$

Standard(s): MGSE9-12.A.CED.2
Create exponential equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.

Review of Percentages: Remember percentages are always out of 100.

To change from a percent to a decimal:

Option 1: Divide by 100 **Option 2:** Move the decimal two places to the left

25% = 0.25 6.5% = 0.065 2% = 0.02 10% = 0.1 3.05% = 0.0305

.02 10 .03.05

Exponential Growth and Decay

Can you tell whether these functions represent growth or decay?

A. $y = 8(4)^x$ G B. $f(x) = 2(5/7)^x$ D C. $h(x) = 0.2(1.4)^x$ G D. $y = ¾(0.99)^x$ D E. $y = ½(1.01)^x$ G

When there is a percent involved in any exponential growth and decay problem, we will use a slightly different equation than $y = ab^x$.

$y = ab^x$

Exponential Growth

- A quantity increases over time

$y = a(1 + r)^t$
where a>0

y = final amount
a = initial amount
r = growth rate (express as decimal)
t = time

(1 + r) represents the growth factor

Exponential Decay

- A quantity decreases over time

$y = a(1 - r)^t$
where a>0

y = final amount
a = initial amount
r = decay rate (express as decimal)
t = time

(1 – r) represents the decay factor

Guided Practice **Finding Growth and Decay Rates**

Identify the following equations as growth or decay. Then identify the initial amount, growth/decay factor, and the growth/decay percent.

a. $y = 3.5(1.03)^t$

Growth/Decay: **growth**

Initial Amount: **3.5**

Growth/Decay Factor: **1.03**

Growth/Decay %: **0.03 = 3%**

$1.03 - 1 = 0.03$
x100

b. $f(t) = 10,000(0.95)^t$

Growth/Decay: D

Initial Amount: 10,000

Growth/Decay Factor: 0.95

Growth/Decay %: 5%

$1 - 0.95$
$= 0.05 \times 100$

c. $y = 2,500(1.2)^t$

Growth/Decay: G

Initial Amount: 2,500

Growth/Decay Factor: 1.2

Growth/Decay %: 20%

$1.2 - 1 = 0.2 \times 100$

Growth and Decay Word Problems

Example 1: The original value of a painting is $1400, and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Growth or Decay: Growth

Starting value (a): 1400

Rate (as a decimal): 0.09

Function: $y=1400(1+0.09)^t$

$y=1400(1.09)^t$

$t=25$

$y=a(1+r)^t$

$y=1400(1.09)^{25}$

$y=\$12,072.31$

Example 2: The cost of tuition at a college is $15,000 and is increasing at a rate of 6% per year. Find the cost of tuition after 4 years.

Growth or Decay: Growth

Starting value (a): 15000

Rate (as a decimal): 0.06

Function: $y=15000(1+0.06)^4$

$y=15000(1.06)^4$

$\$18,937.15$

Example 3: The value of a car is $18,000 and is depreciating at a rate of 12% per year. How much will your car be worth after 10 years?

Growth or Decay: Decay

Starting value (a): 18,000

Rate (as a decimal): 0.12

Function: $y=18000(1-0.12)^{10}$

$y=18000(0.88)^{10}$

$y=\$5,013.02$

Example 4: A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. What is the bungee jumper's height at the top of the 5th bounce?

Growth or Decay: Decay

Starting Value: 500

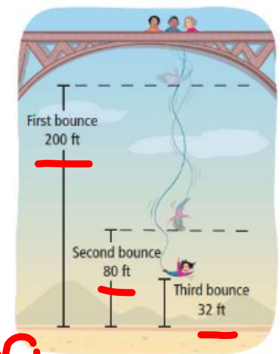
Rate (as a decimal): 0.4

Function: $y=500(0.4)^x$

$y=ab^x$

$b=\frac{80}{200}=0.4$

$a=\frac{200}{0.4}=500\text{ft}$



Day 3 Class Work: Applications - Growth & Decay Assignment **Practice**

Directions: Label if the equation represents growth or decay. Then determine the growth/decay factor and growth/decay rate. Remember to write your rate as a percentage.

1) $y = 10(1.35)^x$ Growth

Growth/Decay Factor: 1.35

Growth/Decay Rate: 35%

$1.35 - 1 = 0.35 \times 100$

2) $y = 742(0.60)^x$ Decay

Growth/Decay Factor: 0.60

Growth/Decay Rate: 40%

$1 - 0.60 = 0.40$

3) $y = (1.04)^x$ growth

Growth/Decay Factor: 1.04

Growth/Decay Rate: 4%

4) $y = 7500(0.42)^x$ Decay

Growth/Decay Factor: 0.42

Growth/Decay Rate: 58%

5) $y = 50(1 + .23)^x$ Growth

Growth/Decay Factor: 1.23

Growth/Decay Rate: 23%

6) $y = 1500(0.925)^x$ Decay

Growth/Decay Factor: 0.925

Growth/Decay Rate: 7.5%

Directions: Create an exponential growth/decay model and use it to solve each problem. Make sure your model problem is in simplified form ($y = ab^x$)

7) A new SUV depreciates at a rate of 23% per year. If the original selling price was $30,000, how much will the vehicle be worth after 4 years?

Model: $y = 30,000(1 - 0.23)^t$ $t = 4$ $y = a(1-r)^t$

$y = 30,000(0.77)^t$

$y = 30,000(0.77)^4$

$\boxed{y = \$10,545.91}$

8) Two bacteria are discovered at the bottom of a shoe. If the bacteria multiply at a rate of 34% per hour, how many bacteria will be present after 48 hours?

Model: $y = 2(1.34)^t$ $t = 48$ $y = a(1+r)^t$

$y = 2(1.34)^{48}$

$y = 2,523,831.3$ Bacteria.

9) The number of student athletes at a local high school is 300 and is increasing at a rate of 8% per year. How many students will be at the school in 5 years?

0.08

Model: $y = 300(1.08)^t$ $t=5$

$y = 300(1.08)^5$

$y = 440.798$

$y = 440$ Students at the School in 5 years.

10) A scientist is creating a mathematical model for the breakdown of caffeine in the human body. According to her current model, caffeine is broken down at a rate of 5% each hour. If a person consumes a sample containing 150 milligrams of caffeine, then how much will remain in 7 hours?

Model: $y = 150(1-0.05)^t$ $t=7$

$y = 150(0.95)^t$

$y = 150(0.95)^7$

$\boxed{y = 104.75\text{mg}}$

11) Riley owns a painting that is valued at $59,000. If the value of the artwork decreases by 5% every year, how much will it be worth in 14 years?

0.05

Model: $y = 59000(1-0.05)^t$ $t=14$

$y = a(1-r)^t$

$y = 59000(0.95)^t$

$y = 59000(0.95)^{14}$

$\boxed{y = \$28,772.82}$

12) Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only 1 bacteria, which can double every hour, how many bacteria will we have by the end of the day?

Model: $y = 1(2)^t$ $t=24$

$y = ab^x$

$y = 1(2)^{24}$

$\boxed{y = 16,777,216 \text{ Bacteria}}$

13) Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds? $t=5$

Model: $y = 128(\frac{1}{2})^t$

$y = 128(\frac{1}{2})^5$

$y = 4$ Players remain after 5 rounds.